3　解答

【例１．１】

＊

$$E[X] = \int_{\alpha}^{\beta} x \cdot \frac{1}{\beta-\alpha} dx = \frac{1}{\beta-\alpha} \cdot \frac{1}{2}[x^2]_{\alpha}^{\beta} = \underline{\frac{1}{2}(\beta+\alpha)}$$

＊

$$E[X^2] = \int_{\alpha}^{\beta} x^2 \cdot \frac{1}{\beta-\alpha} dx = \frac{1}{\beta-\alpha} \cdot \frac{1}{3}[x^3]_{\alpha}^{\beta} = \frac{1}{3} \cdot \frac{\beta^3-\alpha^3}{\beta-\alpha} = \frac{1}{3}(\alpha^2+\alpha\beta+\beta^2)$$

であるので

$$V[X] = E[X^2] - (E[X])^2 = \frac{1}{3}(\alpha^2+\alpha\beta+\beta^2) - \frac{1}{4}(\alpha^2+2\alpha\beta+\beta^2) = \frac{1}{12}(\alpha^2+\beta^2-2\alpha\beta) = \underline{\frac{1}{12}(\beta-\alpha)^2}$$

【問題２．１】

１．

$$E[X^k] = \frac{1}{2}\int_0^2 x^k dx = \frac{1}{2} \cdot \frac{1}{k+1}[x^{k+1}]_0^2 = \frac{2^{k+1}}{2(k+1)} = \underline{\frac{2^k}{k+1}}$$

２．

$$E[Y^k] = \frac{1}{2}\int_{-1}^1 x^k dx = \frac{1}{2} \cdot \frac{1}{k+1}[x^{k+1}]_{-1}^1 = \frac{1-(-1)^{k+1}}{2(k+1)}$$

であるので

$$\mu = E[Y] = 0$$
$$E[Y^2] = \frac{2}{6} = \frac{1}{3}$$
$$\sigma^2 = E[Y^2] - E[Y]^2 = \frac{1}{3}$$
$$E[Y^3] = 0$$
$$E[Y^4] = \frac{1}{5}$$

これより

$$\text{歪度} = \frac{E[Y^3]}{\sigma^3} = \underline{0}$$
$$\text{尖度} = \frac{E[Y^4]}{\sigma^4} = \underline{\frac{9}{5}}$$